A Reduced Complexity Channel Estimator for Space-Frequency Block Coded OFDM Systems

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Abstract

In this paper, channel estimation for space-frequency block coded (SFBC) orthogonal frequency division multiplexing (OFDM) system is considered. By assuming that the consecutive subcarrier of the channel is same, we will develop the channel estimator based on the criterion of the minimum mean square error (MMSE). In addition, we will focus on the reduction of the complexity. Simulation results are given to show the proposed channel estimator is accurate and effective in multipath fading channel.

I. Introduction

In recent years, there has been a lot of attention concentrated on Orthogonal Frequency Division Multiplexing (OFDM). In OFDM systems, the entire channel is divided into many narrow parallel subchannels, thereby the data symbol duration is increased. This is why intersymbol interference (ISI) caused by multipath delay spread is eliminated in OFDM systems. However, due to the dispersive fading property of the wireless environment, there exists higher error probability for those subchannels in deep fades. Therefore, combining OFDM with transmit diversity is an effective technique to combat fading in mobile wireless channel [1]. A space frequency block coded (SFBC) OFDM among other schemes is an effective transmit diversity technique for application where the normalized Doppler frequency is large [2].

In OFDM systems with transmit diversity, the channel state information (CSI) is needed in order to coherently decode the transmitted signal. If CSI is not accurate, the system performance will be seriously degraded. Hence, the channel estimation is indispensable part of the system. Various channel estimation methods for OFDM systems with transmit diversity have been introduced in [3], [4]. However, most of theses are not suitable for SFBC-OFDM. Therefore we will propose a new channel estimator for SFBC-OFDM in time-varying multipath fading channels.

II. System Description

In this section, we briefly derive a mathematical model for SFBC-OFDM systems, as well as the mobile wireless channel statistics of the OFDM systems.

A. Space-Frequency Block Coded OFDM

Consider a $K$ -subcarrier OFDM system with $N_T$ transmit antennas and $N_R$ receive antennas. At a transmission time slot $n$, the space-frequency encoder transforms a signal data block into $N_T$ parallel space-frequency block coded data blocks. Each SFBC data block is then modulated by a $K$-point inverse discrete Fourier transform (IDFT) into an OFDM block. A length $L$ cyclic prefix is added to each OFDM block to avoid the inter-symbol interference due to the multipath fading. Each OFDM block with cyclic prefix is transmitted through a frequency selective fading channel at each transmit antenna simultaneously. At each receiver, the cyclic prefix is first removed from the received signal. Each demodulator performs a $K$-point discrete Fourier transform (DFT) on the OFDM block. The reconstructed signal of the $n$ -th time slot and the $k$ -th subcarrier at the $j$ -th receive antenna is given by

$$ r_j[n,k] = \sum_{i=1}^{N_T} H_{ij}[n,k] s_i[n,k] + v_j[n,k] \quad (1) $$

where $H_{ij}[n,k]$ denotes the channel frequency response for the $k$ -th subcarrier at time $n$, corresponding to the channel between the $i$ -th transmit antenna and the $j$ -th receive antenna. Likewise, $s_i[n,k]$ denotes the symbol for the $k$ -th subcarrier at time $n$ is transmitted from the $i$-th transmit antenna, $v_j[n,k]$ denotes the additive white Gaussian noise with zero mean and variance $\sigma^2$, at the $j$ -th receive antenna for the $n$ -th time slot.
and the $k$-th subcarrier. We assume the number of the transmit antenna $N_T$ and the receive antenna $N_R$ of the OFDM systems that is analyzed the rest of this paper is two and one respectively. Therefore we will omit the subscript of the receive antenna for simplicity. However, the presented techniques can be easily extended to the case with multiple antennas more than two. A simplified block diagram for the proposed systems is shown in Fig. 1.

B. Space-Frequency Block Encoder

A new transmit diversity scheme using two transmit antennas was studied by Alamouti [5]. This scheme was generalized to an arbitrary number of transmit antennas in [6] and was referred to as space-time block coding. Space-time block codes provide full spatial diversity and are amenable to a very simple decoding algorithm based only on linear processing at the receiver. The space-time block coded (STBC) OFDM encoding method is as follows. The symbol transmitted from antenna one is denote by $s_1$ and from antenna two by $s_2$. These symbols are transmitted simultaneously. During the next time slot the symbol $-s_2^*$ is transmitted from antenna one, and the symbol $s_1^*$ is transmitted from antenna two where $*$ is the complex conjugate operation. The channel is assumed to be quasi-static (i.e., it remains constant over the duration of a few OFDM blocks.) The SFBC-OFDM systems are different from STBC-OFDM system. In SFBC-OFDM systems the codes are divided in two subchannels, or subcarriers. At a given subcarrier, the symbol $s_1$, and $s_2$ are transmitted from the two antennas respectively. At the next subcarrier the symbol $-s_2^*$ and $s_1^*$ are transmitted. The structures of STBC and SFBC are shown in Fig. 2. In SFBC-OFDM, the adjacent channel subcarrier responses are correlated. Generally the smaller ratio of the channel delay spread to the OFDM block interval, the more correlated the adjacent subcarrier responses are. In this analysis, the adjacent subcarrier responses are assumed to be same.

![Fig. 2. The structures of STBC and SFBC.](image)

C. Statistics of Mobile Wireless Channel

In our analysis we consider a Rayleigh fading multipath channel model between the $i$-th transmit antenna and the receive antenna is given by

$$h_i(t;\tau) = \sum_{m=0}^{M-1} a_{m,i}(t) \delta(t - \tau_{m,i})$$

where $a_{m,i}(t)$ denotes the $m$-th path complex gain, $\tau_{m,i}$ represents the corresponding path delay time and $M$ is the total number of paths. Each path gain is described by wide-sense stationary (WSS) and zero mean complex Gaussian process generated using the Jake’s spectrum. For different $m$ and $i$, those are independent. The channel frequency response can be expressed as

$$H_i(f,\tau) = \sum_{m=0}^{M-1} a_{m,i}(\tau) \exp(-j2\pi f\tau_{m,i})$$

where

$$H_i(n,k) = H_i(nT_s, k\Delta f) = \sum_{m=0}^{M-1} a_{m,i}(nT_s) \exp(-j2\pi k\tau_{m,i}/T_s)$$

The channel gain for the $k$-th subcarrier of time slot $n$ is

$$H_i(n,k) = H_i(nT_s, k\Delta f) = \sum_{m=0}^{M-1} a_{m,i}(nT_s) \exp(-j2\pi k\tau_{m,i}/T_s)$$

where $T_s = T_b + T_g$ is the OFDM block duration including guard interval, $T_b$ is the data duration of the OFDM block and $T_g$ is the duration of the cyclic prefix. $\Delta f = 1/T_b$ is the subcarrier spacing. In our analysis, the cyclic prefix is assumed to be longer than the largest delay time of the multipath channel.

III. Reduced Complexity Channel Estimation

In this section, we will propose a reduced complexity channel estimators based on MMSE criterion using both frequency and time correlation. This channel estimator structure consists of the three parts.

A. Pilot symbol Pattern

We first present the pilot symbol patterns. The pilot symbols are placed at specified intervals in the OFDM frequency-time signal space, so that the frequency and time correlation of the channel is used for the channel estimation. The number of pilots to use is a trade-off between data rate and channel estimation performance. However, there exist both minimum subcarrier spacing and minimum symbol spacing between pilots. To determine these, we need to find the Doppler spread $B_d$ and the maximum delay spread $\tau_{\max}$ of the wireless channel under consideration. Hence, the requirements for the pilot spacings in frequency and time axis $I_f$ and $I_t$ are

$$I_f < 1/\tau_{\max}, \quad I_t < 1/B_d$$

In addition, the pilot pattern is allocated as the SFBC scheme in order to distinguish a channel response from the other channel responses from the different transmit antennas. This pilot pattern is shown in Fig. 3.

![Fig. 3. The pilot patterns for transmit antenna 1 and 2.](image)

B. Reduced Complexity Channel Estimator

First, the simple estimates, $\tilde{H}_i,p(n,2k)$ is obtained using the orthogonality of the SFBC. As previously stated, two adjacent channel frequency responses are assumed to be same, and
\( p_1(n,2k) = p_2(n,2k) = p_1(n,2k+1) = p \), \( p_2(n,2k+1) = -p \).
\[
r(n,2k) = H_1(n,2k) p(n,2k) + H_2(n,2k) p(n,2k+1) + v(n,2k)
\]
\[
r(n,2k+1) = H_1(n,2k+1) p(n,2k) + H_2(n,2k+1) p(n,2k+1) + v(n,2k+1)
\]
where \( p_i(n,k) \) denotes the pilot symbol transmitted from \( i \)-th transmit antenna for \( k \)-th subcarrier at time \( n \).

Using a simple linear operation of the above two equations, we can get the simple estimates for the \( 2k \)-th subcarrier.
\[
\tilde{H}_{1,p}(n,2k) = \frac{r(n,2k) + r(n,2k+1)}{2p}
\]
\[
\tilde{H}_{2,p}(n,2k) = \frac{r(n,2k) - r(n,2k+1)}{2p}
\]
The linear calculation for AWGN in (8), (9) reduces the variance of the noise by half.

The channel estimation problem is now to find the channel estimates of the entire subcarrier as a linear combination of the simple estimates \( \tilde{H}_{i,p}(n,2k) \). The 2-D MMSE filter using full correlation is an optimal method in order to minimize the MSE. Since 2-D filters in general tend to have a large computational complexity, the method using two 1-D filters is a good alternative. First, a 1-D filter is applied in the frequency direction. Thereafter, a 1-D filter is applied in the time direction. However, if we use all simple estimates \( \tilde{H}_{i,p}(n,2k) \) as estimating in the frequency direction, the complexity reduction is negligible. Hence the \( N_f \)

simple estimates, which are nearest to the channel frequency responses to estimate, are only used as shown in Fig. 4. Since the channel frequency responses far away from the one to estimate are weakly correlated, they do not contribute much to the estimate. By excluding them, the complexity goes down while the performance is almost the same.

The MMSE for this problem is given by
\[
H_{\text{MMSE}}(n) = R_{HH,F}^{-1} \tilde{H}(n) = R_{HH,F}^{-1} (R_{HH,F} + \frac{1}{SNR_{\text{new}}} I)^{-1} \tilde{H}(n)
\]
where \( R_{HH,F} = E[\tilde{H}(n)\tilde{H}(n)^H] \),
\( R_{HH,F} = E[\tilde{H}(n)\tilde{H}(n)^H] \),
\( SNR_{\text{new}} = p^2/\sigma_n^2 = 2p^2/\sigma_n^2 \),
\( \tilde{H}(n) \) denotes the true channel frequency response vector of the subcarriers that we want to estimate. The vector \( \tilde{H}(n) \) is composed of the nearest \( N_f \) subcarriers that are estimated by (8), (9).

IV. Simulation and Performance

In this section we will present some simulation results to show the performance of the proposed MMSE channel estimator for SFBC-OFDM systems. In addition, performance of SFBC OFDM with perfect CSI and 2-D MMSE estimator using full correlation are given to provide some comparison of the proposed estimator. Before presenting the simulation results, we first describe the parameters of the simulated SFBC-OFDM systems.

A. Parameters of SFBC OFDM systems

In our simulation, the entire channel bandwidth, 800 kHz, is divided into 256 subchannels. The eight subchannels on each end are used as guard tones, and the rest are used to transmit data is modulated by QPSK. A cyclic prefix of 16 samples is appended to the OFDM symbol. Thus the total OFDM block length is 340 \( \mu \text{s} \). The channel has 4 impulses, an exponentially decaying power-delay profile \( p_1(t) = C \exp[-\tau_{m,i}/\tau_{rms}] \) and Doppler...
frequencies of 10 and 100 Hz are used. \( N_f \) and \( N_t \) are set to five and two, respectively. The number of multiplications per subcarrier is 3.25.

### B. Simulation Results

The MSE performance of channel estimation methods of proposed in this paper and 2-D MMSE method using full correlation for different Doppler frequencies is shown in Fig. 5. It shows that the MSE of proposed method is close to 2-D MMSE method. Besides the proposed channel estimator is preferable to the 2-D MMSE channel estimator at low SNR. We will guess the reason that the more elements that has severe noise are used to 2-D MMSE estimation. Thus, the proposed method is demonstrated to be efficient nevertheless the complexity is significantly reduced. The performances of both methods for high Doppler frequency degrade in comparison with for low Doppler frequency. However, the difference of the two methods is negligible.

![Fig. 5. MSE for the proposed method and 2-D MMSE method.](image)

Fig. 5. MSE for the proposed method and 2-D MMSE method.

Fig. 6. shows the BER performance of the channel estimators proposed in this paper and 2-D MMSE for different Doppler frequencies, together with the performance of the case with perfect CSI. This result shows that there is some performance degradation in the proposed method in comparison with the case of perfect CSI. This degradation is decreased if the more pilot symbol is used. However, the BER curves of the proposed method are close to that of 2-D MMSE method. Besides for high Doppler frequency the performance of the proposed method is also close to the case of 2-D MMSE. The proposed method in this paper is an effective method for fast fading channel, while it has very low complexity.

![Fig. 6. BER for the proposed method and 2-D MMSE method.](image)

Fig. 6. BER for the proposed method and 2-D MMSE method.

### V. Conclusions

In this paper, the channel estimator for SFBC-OFDM systems has been proposed. This method is based on 2-D MMSE filter. However, in order to reduce the complexity, it uses two 1-D MMSE filters are applied in frequency and time direction, respectively. Since the nearest some simple estimates are only used in frequency direction MMSE filter, the complexity is further reduced. Although the Doppler frequency of the channel is high, simulation results demonstrate the good performance of the proposed channel estimator.

### References


